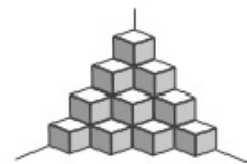


1. The tower shown is made of congruent cubes stacked on top of each other. Some of the cubes are not visible. How many cubes in all are used to form the tower?



2. Let  $a, b$  and  $C$  be positive integers such that  $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ . If the greatest common divisor of  $a, b$  and  $c$  is 1, then prove that  $b + c$  must be a perfect square.

3. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip,  $abc$  miles were displayed on the odometer, where  $abc$  is a 3-digit number with  $a \geq 1$  and  $a + b + c \leq 7$ . At the end of the trip, where the odometer showed  $cba$  miles.



What is  $a^2 + b^2 + c^2$ ?

4. The number 2017 is prime. Let  $S = \sum_{k=0}^{62} \binom{2014}{k}$ . What is the remainder when  $S$  is divided by 2017?

(\*) 5. There are 100 prisoners all in solitary confinement. The warden of the prison decides to give them all a test. He will randomly choose prisoners one at a time to go into a room with only a lightbulb. The prisoner can either turn the bulb on if it is off, off it is on, or do nothing and leave it as it was. The goal of the prisoners is eventually have someone say with absolute certainty "All one hundred prisoners have been in this room at least once before." If he is correct, all 100 go free, otherwise all 100 are killed, so whoever says it must be 100% sure that he is correct. The warden gives them one night to plan how to pass the wardens test.



Assumptions: Everyone knows that the lightbulb is off at the beginning. Anyone that goes in only knows whether the lightbulb was on or off when enter, what they do to the bulb (change it or nothing) and how many times they themselves have been in the room, nothing else. Since everyone is in solitary, no one knows how fast time is passing, and the time in between the warden's choice of prisoner is arbitrary. There is nothing else to know or do in the room besides the status of the lightbulb, and there is no way the prisoners can communicate outside of the one night of planning and the lightbulb. What is the strategy to beat the warden?

- Solutions to problems marked with the (\*) are to be **carefully written up at home and submitted** at the following session (Sept 26, 2014).
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