1. Some faces of a certain convex polyhedron are regular pentagons and others are regular hexagons. How many of these faces are pentagons?

2. Is there an infinite sequence of positive integers $a_1 < a_2 < a_3 < ...$ such that for any integer n the sequence $\{a_i + n\}$ contains only finitely many primes?

3. **(*)** $2n + 1$ knights stand in the battlefield in such a way so that the distances between every two of them are different. Each of them shoots and kills the closest knight to him (all the knights shoot simultaneously). Show that at least one knight survives.

4. On the round table of radius R there are n non-overlapping coins, in such a way that no more coins can be added to the table. Assume all the coins have radius r. Show that

$$\frac{1}{2}\left(\frac{R}{r} - 1\right) \leq \sqrt{n} \leq \frac{R}{r}$$

5. 100 pirates found 100 gold coins. The leader(who happens to be the oldest pirate) suggests how to divide the treasure, and then a democratic vote is held. If his proposal is supported by at least 50 percent of the pirates, the treasure is divided according to his proposal. Otherwise he is executed and the next oldest pirate is appointed as a new leader and has a duty to suggest how to divide the treasure. The process is repeated until the agreement is achieved. The priorities of the pirates are to save their own life first, then to get as much gold as possible, and then to kill as many other pirates as possible. What would you do if you were the leader?

- Solutions to problems marked with the **(*)** are to be **carefully written up at home and submitted** at the following session (Oct 24, 2014).
- Follow us at **uicmop.wordpress.com** or at **UIC MOP** on Facebook/Google+